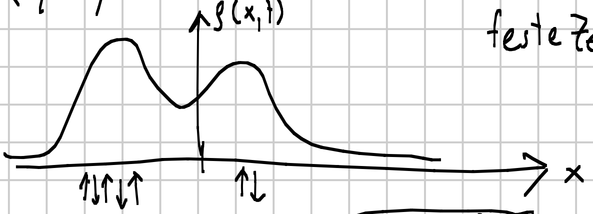
HM4 ZÜ Blatt 8

Notiztitel 12.06.2008

25. $\rho(x,t)$ Verkehrsdichte. Geschwindigkeit v

feste Zeit t



$$\boxed{\rho_t + v\rho_x = f(x,t)}$$

inhomogene lineare pDgl 1. Ordnung mit konst. Koeff.

Bewegtes Koordinatensystem: $y = x - vt$.

$\begin{pmatrix} x \\ t \end{pmatrix} \mapsto \begin{pmatrix} y \\ t \end{pmatrix} = \begin{pmatrix} x - vt \\ t \end{pmatrix}$

$$R(y,t) = \rho(y+vt,t)$$

$$\Rightarrow R_y(y,t) = \rho_x(y+vt,t)\cdot 1 + 0$$

$$R_t(y,t) = \rho_x(y+vt,t)\cdot v + \rho_t(y+vt,t)\cdot 1 = f(y+vt,t)$$

$$\underline{R_t(y,t) = f(y+vt,t)}$$

$$\Rightarrow R(y,t) = R(y,0) + \int_0^t R_t(y,s)\,ds$$

$$= \rho(y,0) + \int_0^t f(y+vs,s)\,ds$$

$$\varrho(x,t) = R(x-vt,t) = \varrho(x-vt,0) + \int_0^t f(x-v(t-s),s)\,ds$$

$y = \alpha x + \beta t$, $\alpha = 1$ $\quad R(y,t) = \varrho(x,t)$

$\partial_t R(y,t) = \partial_x \varrho(y-\beta t,t)\cdot(-\beta) + \partial_t \varrho(y-\beta t,t)$, $\beta = -v$

(a) homogener Fall $f(x,t) = 0$. $\varrho(x,0) = \varrho_0(x)$

$\Rightarrow$ Lsg: $\varrho(x,t) = \varrho(x-vt,0) = \varrho_0(x-vt)$.

(b) inhomogen $f(x,t) = \dfrac{1}{1+x^2}$, $\varrho_0(x) = 0$

$$\varrho(x,t) = 0 + \int_0^t f(x-vt+vs,s)\,ds = \int_0^t \frac{1}{1+(x-vt+vs)^2}\,ds \underset{dy = v\,ds}{\overset{y = x-vt+vs}{=}}$$

$$= \frac{1}{v}\int_{x-vt}^{x} \frac{1}{1+y^2}\,dy = \frac{1}{v}\left(\arctan(x) - \arctan(x-vt)\right)$$

(c) inhomogen $f(x,t) = \dfrac{1}{1+x^2+v^2t^2}$, $\varrho_0(x) = 0$

$$\varrho(x,t) = 0 + \int_0^t \frac{1}{1+(x-vt+vs)^2+v^2s^2}\,ds =$$

$$= \int_0^t \frac{1}{2\left(vs+\frac{1}{2}(x-vt)\right)^2 + \frac{1}{2}(x-vt)^2+1}\,ds \underset{dy = v\,ds}{\overset{y = \frac{1}{2}(x-vt)+vs}{=}}$$

$$= \int_{\frac{1}{2}(x-vt)}^{\frac{1}{2}(x+vt)} \frac{1}{2y^2 + \left(1+\frac{1}{2}(x-vt)^2\right)} \frac{dy}{v} \qquad z = \frac{2y}{\sqrt{2+(x-vt)^2}} =$$

$$= \frac{1}{v} \int_{\frac{x-vt}{\sqrt{2+(x-vt)^2}}}^{\frac{x+vt}{\sqrt{2+(x-vt)^2}}} \frac{1}{\sqrt{2+(x-vt)^2}} \frac{1}{1+z^2}\, dz$$

$$= \frac{1}{v\sqrt{2+(x-vt)^2}} \left(\arctan \frac{x+vt}{\sqrt{2+(x-vt)^2}} - \arctan \frac{x-vt}{\cancel{v}\sqrt{2+(x-vt)^2}} \right).$$